

Semileptonic and Exclusive Rare B Decays

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The exclusive rare decay $B \rightarrow K^*\gamma$ takes place in a region of maximum recoil, $q^2 = 0$, posing a problem for nonrelativistic quark models which are usually thought to be most reliable at zero recoil. The Bauer–Stech–Wirbel (BSW) model, formulated in the infinite-momentum-frame (IMF) formalism, is designed to work at $q^2 = 0$. We show in this model that the ratio relating the decay $B \rightarrow K^*\gamma$ and the q^2 -spectrum of the semileptonic decay $B \rightarrow \rho e\bar{\nu}$, becomes independent of the wave function in the SU(3) flavor symmetry limit. We show that this feature is also true in relativistic quark models formulated in the IMF or light-cone formalism, if the b quark is infinitely heavy. In fact, these relativistic models, which have a different spin structure from the BSW case, reduce to the BSW model in the heavy b -quark limit. A direct measurement of the q^2 -spectrum of the semileptonic decay can therefore provide accurate information for the exclusive rare decay.

1 Introduction

The first experimental observation of the exclusive decay $B \rightarrow K^*$ has been reported from the CLEO collaboration [1] which gives a branching ratio of $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. Theoretically, this rare decay is not well understood, as there is still an uncertainty of a factor of about 10 in the branching ratio depending on the way the large recoil of the K^* is handled [2] in the form factors. Burdman and Donoghue [3] pointed out that the heavy-quark symmetry together with the SU(3) flavor symmetry could relate the rare decay $B \rightarrow K^*\gamma$ to a measurement of the semileptonic decay $B \rightarrow \rho e \bar{\nu}$, independent of the form factors. However, the relation is only valid at a single point in the Dalitz plot, a point where the semileptonic decay vanishes, so that there would still be a large uncertainty in such a measurement. In a recent paper [4] O'Donnell and Tung studied the possibility of relating the decay $B \rightarrow K^*\gamma$ to the q^2 -spectrum of the semileptonic decay $B \rightarrow \rho e \bar{\nu}$. They showed that the ratio \mathcal{I} (to be defined below), which relates the two decays at $q^2 = 0$, is quite insensitive to different models and the wave functions. Since the q^2 -spectrum of $B \rightarrow \rho e \bar{\nu}$ does not vanish at $q^2 = 0$, a direct measurement of the spectrum at this point can therefore provide quite accurate information for $B \rightarrow K^*\gamma$. An application of this was made in Ref. [5].

In this letter we first study the ratio \mathcal{I} in the BSW model [6]. We find that in the SU(3) flavor symmetry limit, this ratio becomes independent of the wave function used in this model. The BSW model is formulated in the IMF formalism and, since $q^2 = 0$ is the maximum recoil region, this seems to be an appropriate frame. The model is not a completely relativistic one, however, in the sense that the spin and orbital parts of the wave function are factorized. There are relativistic quark models formulated in the IMF or light-cone formalism [7, 8] for which the spin and orbital parts of the wave function are not factorized. We show that in such models, the ratio \mathcal{I} is also independent of the wave function, if the b quark is infinitely heavy.

In fact the relativistic quark models become equivalent to the BSW model in the heavy b -quark limit. This result does not depend on assuming a heavy s quark.

2 $B \rightarrow K^* \gamma$ versus $B \rightarrow \rho e \bar{\nu}$

We define the form factors in $B \rightarrow K^* \gamma$ and $B \rightarrow \rho e \bar{\nu}$ by

$$\begin{aligned} \langle V(p_V, \epsilon) | \bar{Q} i \sigma_{\mu\nu} q^\nu b_R | B(p_B) \rangle &= f_1(q^2) i \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p_B^\lambda p_V^\sigma \\ &+ \left[(m_B^2 - m_V^2) \epsilon_\mu^* - (\epsilon^* \cdot q)(p_B + p_V)_\mu \right] f_2(q^2) \\ &+ (\epsilon^* \cdot q) \left[(p_B - p_V)_\mu - \frac{q^2 (p_B + p_V)_\mu}{(m_B^2 - m_V^2)} \right] f_3(q^2), \end{aligned} \quad (1)$$

and

$$\begin{aligned} \langle V(p_V, \epsilon) | \bar{Q} \gamma_\mu \gamma_5 b | B(p_B) \rangle &= \frac{2V(q^2)}{m_B + m_V} i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^\alpha p_V^\beta + 2m_V \frac{(\epsilon^* \cdot p_B)}{q^2} q^\mu A_0(q^2) \\ &+ \left[(m_B + m_V) \epsilon^{*\mu} A_1(q^2) - \frac{(\epsilon^* \cdot p_B)}{m_B + m_V} (p_B + p_V)^\mu A_2(q^2) - 2m_V \frac{(\epsilon^* \cdot p_B)}{q^2} q^\mu A_3(q^2) \right]. \end{aligned} \quad (2)$$

The branching ratio for the exclusive $B \rightarrow K^* \gamma$ to the inclusive $b \rightarrow s \gamma$ processes can be written in terms of $f_1(0)$ and $f_2(0)$ at $q^2 = 0$, as [9, 10]

$$R(B \rightarrow K^* \gamma) = \frac{\Gamma(B \rightarrow K^* \gamma)}{\Gamma(b \rightarrow s \gamma)} \cong \frac{m_b^3 (m_B^2 - m_{K^*}^2)^3}{m_b^3 (m_b^2 - m_s^2)^3} \frac{1}{2} \left[|f_1(0)|^2 + 4|f_2(0)|^2 \right]. \quad (3)$$

In most models $f_2(0) = \frac{1}{2} f_1(0)^*$. Although there is now only one form factor to calculate in Eq. (3), this is still a controversial model-dependent calculation [2].

Burdman and Donoghue [3] give a method of relating $B \rightarrow K^* \gamma$ to the semileptonic process $B \rightarrow \rho e \bar{\nu}$ using the static b -quark limit and SU(3) flavor symmetry, independent of the model dependence of the form factors. Their main result is

$$\Gamma(B \rightarrow K^* \gamma) \left(\lim_{q^2 \rightarrow 0, \text{curve}} \frac{1}{q^2} \frac{d\Gamma(B \rightarrow \rho e \bar{\nu})}{dE_\rho dE_e} \right)^{-1} = \frac{4\pi^2}{G_F^2} \frac{|\eta|^2}{|V_{ub}|^2} \frac{(m_B^2 - m_{K^*}^2)^3}{m_B^4}. \quad (4)$$

* In the heavy b quark limit [4] this relation is exact.

Here η represents the QCD corrections to the decay $b \rightarrow s\gamma$ and the word “curve” denotes the region in the Dalitz plot where $q^2 = 4E_e(m_B - E_\rho - E_e)$. Their method replaces the uncertainty in the calculation at large recoil ($q^2 = 0$) of the $B \rightarrow K^*$ form factors by making a direct measurement of $B \rightarrow \rho e \bar{\nu}$. The problem is that the semileptonic decay vanishes at the $q^2 = 0$ point on the “curve,” which is why this kinematic factor is divided out in Eq. (4). This means that experimentally there should be no events at that point and very few in the neighborhood, making it a very difficult measurement.

To avoid this O’Donnell and Tung [4] studied instead the q^2 -spectrum of the semileptonic decay $B \rightarrow \rho e \bar{\nu}$. In this case

$$R(B \rightarrow K^* \gamma) \left(\frac{d\Gamma(B \rightarrow \rho e \bar{\nu})}{dq^2} \Big|_{q^2=0} \right)^{-1} = \frac{192\pi^3}{G_F^2} \frac{1}{|V_{ub}|^2} \frac{(m_B^2 - m_{K^*}^2)^5}{(m_B^2 - m_\rho^2)^5} \frac{(m_B - m_\rho)^2}{(m_B - m_{K^*})^2} \frac{m_b^3}{(m_b^2 - m_s^2)^3} |\mathcal{I}|^2, \quad (5)$$

where \mathcal{I} is defined as

$$\mathcal{I} = \frac{(m_B + m_\rho)}{(m_B + m_{K^*})} \frac{f_1^{B \rightarrow K^*}(0)}{A_3^{B \rightarrow \rho}(0)}. \quad (6)$$

The advantage here is that the q^2 -spectrum does not vanish at $q^2 = 0$. The disadvantage is that in taking the ratio we do not have in general the simple cancellation of form factors. However, as we will see, in a number of models the ratio \mathcal{I} is still relatively free of uncertainties.

3 The BSW model

The BSW model, formulated in the IMF formalism, is designed to work at $q^2 = 0$ where the ratio \mathcal{I} of Eq. (6) is defined. In this model, the form factors $f_1(0)$ and $A_3(0)$ are given by

$$A_3(0) = f_1(0) = \int dx \, d^2\mathbf{k}_\perp \phi_V^*(x, \mathbf{k}_\perp) \phi_B(x, \mathbf{k}_\perp), \quad (7)$$

where $\phi(x, \mathbf{k}_\perp)$ is the wave function. Thus, the ratio \mathcal{I} in Eq. (6) becomes 1 if we use the SU(3) flavor symmetry for ρ and K^* . This is independent of the wave functions used.

In the limit when both $m_b \rightarrow \infty$ and $m_Q \rightarrow \infty$, the heavy quark symmetry [11] gives $A_3(0) = f_1(0)$ for $B(\bar{q}b) \rightarrow V(\bar{q}Q)$ transitions; this equality is not expected to be true for arbitrary m_b and m_Q . In the BSW model this equality between $f_1(0)$ and $A_3(0)$ for arbitrary m_b and m_Q comes from the factorization of the spin and orbital parts of the wave function. The total wave function $\Psi^{J,J_3}(\mathbf{p}_1, \mathbf{p}_2, \lambda_1, \lambda_2)$ has the simple form

$$\Psi^{J,J_3}(\mathbf{p}_1, \mathbf{p}_2, \lambda_1, \lambda_2) = \chi_{\lambda_1}^\dagger S^{J,J_3} \chi_{\lambda_2} \phi(x, \mathbf{k}_\perp) , \quad (8)$$

where $\lambda_{1,2}$ are the spin indices of the quarks and χ is the Pauli spinor. The relation between the coordinates $\mathbf{p}_{1,2}$ and (x, \mathbf{k}_\perp) is given in Eq. (11) below. For the pseudoscalar and vector mesons, the spin wave functions are given by the SU(2) relations

$$S^{0,0} = \frac{i\sigma_2}{\sqrt{2}} , \quad S^{1,\pm 1} = \frac{1 \pm \sigma_3}{2} , \quad S^{1,0} = \frac{\sigma_1}{\sqrt{2}} . \quad (9)$$

Hence, there is a spin symmetry in the model which gives simple relations among the form factors. For example, the form factors $A_1(0)$ and $V(0)$ are also related [6].

4 Relativistic quark models with IMF formalism

Due to its treatment of the quark spins, the BSW model is not a completely relativistic quark model. A method to obtain a relativistic quark model using the IMF or light-cone formalism was developed quite a long time ago [7, 8] and there have been many applications [7, 8, 12, 13, 14]. In [8] and [12], the model was applied to calculate electromagnetic form factors of pions and nucleons. In [13] and [14] similar models were used to study weak decays of mesons and baryons.

We describe a ground-state $Q\bar{q}$ meson V in the infinite momentum frame by

$$|V(P, J_3, J)\rangle = \int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \delta(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2) \sum_{\lambda_1, \lambda_2} \Psi^{J, J_3}(\mathbf{P}, \mathbf{p}_1, \mathbf{p}_2, \lambda_1, \lambda_2) |Q(\lambda_1, \mathbf{p}_1) \bar{q}(\lambda_2, \mathbf{p}_2)\rangle , \quad (10)$$

where $\mathbf{P} = P\mathbf{e}_z$, $P \rightarrow \infty$ and the quark coordinates are

$$p_{1z} = x_1 P, p_{2z} = x_2 P, x_1 + x_2 = 1, 0 \leq x_{1,2} \leq 1, \\ \mathbf{p}_{1\perp} = \mathbf{k}_\perp, \mathbf{p}_{2\perp} = -\mathbf{k}_\perp. \quad (11)$$

Rotational invariance of the wave function for states with spin J and zero orbital angular momentum requires the wave function to have the form [12, 13] (with $x = x_1$)

$$\Psi^{J, J_3}(\mathbf{P}, \mathbf{p}_1, \mathbf{p}_2, \lambda_1, \lambda_2) = R^{J, J_3}(\mathbf{k}_\perp, \lambda_1, \lambda_2) \phi(x, \mathbf{k}_\perp), \quad (12)$$

where $\phi(x, \mathbf{k}_\perp)$ is even in \mathbf{k}_\perp and

$$R^{J, J_3}(\mathbf{k}_\perp, \lambda_1, \lambda_2) = \sum_{\lambda, \lambda'} \langle \lambda_1 | R_M^\dagger(\mathbf{k}_\perp, m_Q) | \lambda \rangle \langle \lambda_2 | R_M^\dagger(-\mathbf{k}_\perp, m_{\bar{q}}) | \lambda' \rangle C^{J, J_3}(\frac{1}{2}, \lambda; \frac{1}{2}, \lambda'). \quad (13)$$

In Eq. (13), $C^{J, J_3}(\frac{1}{2}, \lambda; \frac{1}{2}, \lambda')$ is the Clebsh-Gordan coefficient and the rotation $R_M(\mathbf{k}_\perp, m_i)$ ($i = Q, \bar{q} = 1, 2$) on the quark spins is the Melosh rotation [15]:

$$R_M(\mathbf{k}_\perp, m_i) = \frac{m_i + x_i M_0 - i\sigma \cdot (\mathbf{n} \times \mathbf{k}_\perp)}{\sqrt{(m_i + x_i M_0)^2 + \mathbf{k}_\perp^2}}, \quad (14)$$

where $\mathbf{n} = (0, 0, 1)$ and

$$M_0^2 = \frac{m_1^2 + \mathbf{k}_\perp^2}{x_1} + \frac{m_2^2 + \mathbf{k}_\perp^2}{x_2}. \quad (15)$$

It's easy to see that if there is no transverse momentum, the rotation becomes an unity matrix:

$$R_M(\mathbf{0}, m_i) = 1. \quad (16)$$

The spin wave function $R^{J,J_3}(\mathbf{k}_\perp, \lambda_1, \lambda_2)$ in Eq. (13) can also be written as

$$\begin{aligned} R^{J,J_3}(\mathbf{k}_\perp, \lambda_1, \lambda_2) &= \chi_{\lambda_1}^\dagger R_M^\dagger(\mathbf{k}_\perp, m_Q) S^{J,J_3} R_M^{\dagger T}(-\mathbf{k}_\perp, m_{\bar{q}}) \chi_{\lambda_2} \\ &= \chi_{\lambda_1}^\dagger U_V^{J,J_3} \chi_{\lambda_2} , \end{aligned} \quad (17)$$

where S^{J,J_3} is defined by

$$S^{J,J_3} = \sum_{\lambda, \lambda'} |\lambda\rangle \langle \lambda'| C^{J,J_3}(\frac{1}{2}, \lambda; \frac{1}{2}, \lambda') . \quad (18)$$

For the pseudoscalar and vector mesons, S^{J,J_3} is given in Eq. (9) and the U^{J,J_3} read

$$\begin{aligned} U^{0,0} &= \frac{\sqrt{2}(x_1 m_2 + x_2 m_1) S^{0,0} - \mathbf{k}_- S^{1,1} - \mathbf{k}_+ S^{1,-1}}{\sqrt{2} d_0} , \\ U^{1,0} &= \frac{\sqrt{2}(\alpha_1 \cdot \alpha_2 + \mathbf{k}_\perp^2) S^{1,0} + (m_1 - m_2 + (x_1 - x_2)M_0)(\mathbf{k}_+ S^{1,-1} - \mathbf{k}_- S^{1,1})}{\sqrt{2} d_1 d_2} , \\ U^{1,1} &= \frac{\sqrt{2}\alpha_1 \cdot \alpha_2 S^{1,1} - \sqrt{2} \mathbf{k}_+^2 S^{1,-1} + (\alpha_1 - \alpha_2)\mathbf{k}_+ S^{1,0} + (\alpha_1 + \alpha_2)\mathbf{k}_+ S^{0,0}}{\sqrt{2} d_1 d_2} , \\ U^{1,-1} &= \frac{-\sqrt{2} \mathbf{k}_-^2 S^{1,1} + \sqrt{2}\alpha_1 \cdot \alpha_2 S^{1,-1} - (\alpha_1 - \alpha_2)\mathbf{k}_- S^{1,0} + (\alpha_1 + \alpha_2)\mathbf{k}_- S^{0,0}}{\sqrt{2} d_1 d_2} , \end{aligned} \quad (19)$$

where

$$\alpha_1 = m_1 + x_1 M_0 , \quad \alpha_2 = m_2 + x_2 M_0 , \quad \mathbf{k}_\pm = \mathbf{k}_x \pm i\mathbf{k}_y , \quad (20)$$

and

$$\begin{aligned} d_0 &= \sqrt{\mathbf{k}_\perp^2 + (x_1 m_2 + x_2 m_1)^2} , \quad d_1 = \sqrt{(m_1 + x_1 M_0)^2 + \mathbf{k}_\perp^2} , \\ d_2 &= \sqrt{(m_2 + x_2 M_0)^2 + \mathbf{k}_\perp^2} . \end{aligned} \quad (21)$$

The normalization condition for the wave function is

$$1 = \int dx d^2 \mathbf{k}_\perp \sum_{\lambda_1, \lambda_2} |\Psi^{J,J_3}(\mathbf{p}_1, \mathbf{p}_2, \lambda_1, \lambda_2)|^2 = \int dx d^2 \mathbf{k}_\perp |\phi(x, \mathbf{k}_\perp)|^2 , \quad (22)$$

where

$$R_M^\dagger(\mathbf{k}_\perp, m_i) R_M(\mathbf{k}_\perp, m_i) = 1 . \quad (23)$$

The matrix element of a B meson decaying to a vector meson V is

$$\begin{aligned}\langle V(p_V, J_3) | \bar{Q} \Gamma b | B(p_B) \rangle &= P \int dx \, d^2 \mathbf{k}_\perp \sum_{\lambda_1, \lambda_2} \frac{\Psi_V^{*1, J_3} \bar{u}_Q \Gamma u_b \Psi_B}{\sqrt{4e_b e_Q}} \\ &= P \int dx \, d^2 \mathbf{k}_\perp \frac{\phi_V^* \phi_B}{\sqrt{4e_b e_Q}} \text{Tr} \left[U_V^{\dagger 1, J_3} U_\Gamma U_B^{0,0} \right] ,\end{aligned}\quad (24)$$

where U_Γ is defined by

$$\bar{u}_Q^i \Gamma u_b^j = \chi_i^\dagger U_\Gamma \chi_j , \quad (25)$$

and e_b and e_Q are energies of the quarks. In Eq. (24), we choose $\mathbf{p}_B = P \mathbf{e}_z$, $P \rightarrow \infty$, and the momentum transfer $q = p_B - p_V$ is given by

$$\mathbf{q}_\perp = 0, \quad q_0 = -q_z = \frac{(m_B^2 - m_V^2)}{4P}, \quad q^2 = 0 . \quad (26)$$

In contrast to Eq. (24), the matrix element in the nonrelativistic treatment of quark spins is given by

$$\langle V(p_V, J_3) | \bar{Q} \Gamma b | B(p_B) \rangle = P \int dx \, d^2 \mathbf{k}_\perp \frac{\phi_V^* \phi_B}{\sqrt{4e_b e_Q}} \text{Tr} \left[S^{\dagger 1, J_3} U_\Gamma S^{0,0} \right] . \quad (27)$$

Using Eq. (24), we obtain new expressions for the form factors $A_3(0)$ and $f_1(0)$

$$\begin{aligned}A_3(0) &= \int dx \, d^2 \mathbf{k}_\perp \phi_V^* \phi_B \frac{T_{A_3}}{D} , \\ f_1(0) &= \int dx \, d^2 \mathbf{k}_\perp \phi_V^* \phi_B \frac{T_{f_1}}{D} ,\end{aligned}\quad (28)$$

where the kinematic terms are

$$\begin{aligned}T_{A_3} &= \left[(m_Q + x M_0^V)(m_{\bar{q}} + (1-x)M_0^V) + \mathbf{k}_\perp^2 \right] (x m_{\bar{q}} + (1-x)m_b) \\ &\quad + (m_Q - m_{\bar{q}} + (2x-1)M_0^V) \mathbf{k}_\perp^2 , \\ T_{f_1} &= (m_Q + x M_0^V) \left[\mathbf{k}_\perp^2 + (x m_{\bar{q}} + (1-x)m_b)(m_{\bar{q}} + (1-x)M_0^V) \right] ,\end{aligned}\quad (29)$$

$$\begin{aligned}D &= \sqrt{\mathbf{k}_\perp^2 + (x m_{\bar{q}} + (1-x)m_b)^2} \\ &\quad \times \sqrt{(m_Q + x M_0^V)^2 + \mathbf{k}_\perp^2} \sqrt{(m_{\bar{q}} + (1-x)M_0^V)^2 + \mathbf{k}_\perp^2} .\end{aligned}\quad (30)$$

Here, M_0^V corresponds to Eq. (15) for the meson V .

One interesting feature of the transformation Eq. (13) is that for the heavy pseudoscalar mesons, such as the B meson, one can write

$$R^{0,0}(\mathbf{k}, \lambda_1, \lambda_2) = \frac{\bar{u}_b(\mathbf{p}_1, \lambda_1) \gamma_5 v_{\bar{q}}(\mathbf{p}_2, \lambda_2)}{\sqrt{2} \sqrt{M_0^2 - (m_b - m_{\bar{q}})^2}}, \quad (31)$$

and for the heavy vector mesons such as the B^* meson

$$R^{1,J_3}(\mathbf{k}, \lambda_1, \lambda_2) = -\frac{\zeta^\mu(J_3) \bar{u}_b(\mathbf{p}_1, \lambda_1) \gamma_\mu v_{\bar{q}}(\mathbf{p}_2, \lambda_2)}{\sqrt{M_0^2 - (m_b - m_{\bar{q}})^2}}. \quad (32)$$

If both the V and B mesons in Eq. (24) are heavy, one can express matrix element in terms of the trace expression (as in the heavy quark effective theory [16, 17])

$$\langle V(v', J_3) | \bar{Q} \Gamma b | B(v) \rangle \propto \text{Tr} \left[\frac{1 + \not{v}'}{2} \not{\epsilon} \Gamma \frac{1 + \not{v}}{2} \gamma_5 \right] \quad (33)$$

The meson wave functions $\phi(x, \mathbf{k}_\perp)$ are model dependent and difficult to obtain; often simple forms are assumed for them. One possibility is a Gaussian type of wave function [8, 18]

$$\phi(x, \mathbf{k}_\perp) = N \exp \left(-\frac{M_0^2}{2\beta^2} \right) = N \exp \left(-\frac{1}{2\beta^2} \left[\frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x} \right] \right). \quad (34)$$

In [12, 13] a slightly different harmonic-oscillator wave function $\eta(\mathbf{k})$ was used,

$$\eta(\mathbf{k}) = N \exp \left(-\frac{\mathbf{k}^2}{2\sigma^2} \right), \quad (35)$$

with the normalization,

$$1 = \int dx \, d^2 \mathbf{k}_\perp |\phi(x, \mathbf{k}_\perp)|^2 = \int d^3 \mathbf{k} |\eta(\mathbf{k})|^2. \quad (36)$$

Here, \mathbf{k}_z is defined by $x_1 M_0 = E_1 + k_z$ with $E_1 = \sqrt{\mathbf{k}_\perp^2 + k_z^2 + m_1^2}$ so that

$$\phi(x, \mathbf{k}_\perp) = \sqrt{\frac{d\mathbf{k}_z}{dx}} \eta(\mathbf{k}). \quad (37)$$

A third possibility is the wave function from a relativistic harmonic oscillator equation [6]

$$\phi(x, \mathbf{k}_\perp) = N \sqrt{x(1-x)} \exp \left(-\frac{m_B^2}{2w^2} \left[x - \frac{1}{2} - \frac{m_b^2 - m_q^2}{2m_B^2} \right]^2 \right) \frac{\exp \left(-\frac{\mathbf{k}_\perp^2}{2w^2} \right)}{\sqrt{\pi w^2}}. \quad (38)$$

The parameters β , σ and ω in Eq. (34), Eq. (35) and Eq. (38) are all of the order of Λ_{QCD} .

In the heavy b -quark limit, it is well known that the distribution amplitude $\int d^2\mathbf{k}_\perp \phi(x, \mathbf{k}_\perp)$ of a heavy meson such as the B meson, has a peak near $x \simeq x_0 = \frac{m_b}{m_B}$. As the b quark mass becomes larger, the width of the peak decreases and x_0 comes closer to 1. For the wave function $\phi(x, \mathbf{k}_\perp)$ itself, one expects a similar picture: $\phi(x, \mathbf{k}_\perp)$ vanishes if $\mathbf{k}_\perp^2 \gg \Lambda_{\text{QCD}}^2$ and peaks as $x \rightarrow 1$. All the three wave functions listed before have this feature. This is easy to understand since $\langle \mathbf{k}_\perp^2 \rangle \sim \Lambda_{\text{QCD}}^2$. Also, the average velocity of the heavy quark equals that of the heavy meson so that the average $x \rightarrow 1$.

The effect of the above feature on the matrix element of the current is that the integrand in Eq. (24) vanishes everywhere in the heavy b -quark limit, except for $x \rightarrow 1$ and small \mathbf{k}_\perp . When $x \rightarrow 1$, for both B and V mesons,

$$xM_0 \rightarrow \infty, \quad (1-x)M_0 \rightarrow 0, \quad (39)$$

and the Melosh rotations become,

$$R_M(\mathbf{k}_\perp, m_{\bar{q}}) \rightarrow \frac{m_{\bar{q}} - i\sigma \cdot (\mathbf{n} \times \mathbf{k}_\perp)}{\sqrt{m_{\bar{q}}^2 + \mathbf{k}_\perp^2}}, \quad R_M(\mathbf{k}_\perp, m_Q) \rightarrow 1, \quad R_M(\mathbf{k}_\perp, m_b) \rightarrow 1. \quad (40)$$

Consequently,

$$U_V^{1,J_3} \rightarrow S^{1,J_3} \frac{m_{\bar{q}} - i\sigma \cdot (\mathbf{n} \times \mathbf{k}_\perp)}{\sqrt{m_{\bar{q}}^2 + \mathbf{k}_\perp^2}}, \quad U_B^{0,0} \rightarrow S^{0,0} \frac{m_{\bar{q}} - i\sigma \cdot (\mathbf{n} \times \mathbf{k}_\perp)}{\sqrt{m_{\bar{q}}^2 + \mathbf{k}_\perp^2}}, \quad (41)$$

and Eq. (24) becomes Eq. (27)

$$\begin{aligned} \langle V(p_V, J_3) | \bar{Q} \Gamma b | B(p_B) \rangle &= P \int dx \, d^2\mathbf{k}_\perp \frac{\phi_V^* \phi_B}{\sqrt{4e_b e_Q}} \text{Tr} \left[U_V^{\dagger 1, J_3} U_\Gamma U_B^{0,0} \right] \\ &\rightarrow P \int dx \, d^2\mathbf{k}_\perp \frac{\phi_V^* \phi_B}{\sqrt{4e_b e_Q}} \text{Tr} \left[S^{\dagger 1, J_3} U_\Gamma S^{0,0} \right]. \end{aligned} \quad (42)$$

We conclude that in the heavy b -quark limit the type of relativistic quark models formulated above reduce to the BSW model. Thus, in the heavy b -quark limit and

in SU(3) flavor symmetry, the ratio $\mathcal{I} = 1$ of Eq. (6) remains true for the type of relativistic quark models [7, 8] presented here.

Numerically, the mass of B meson is heavy enough to make the relativistic model very close to the BSW-type model. If the SU(3)-flavor symmetry is preserved in the wave functions (with possible breaking coming from the masses) then $\mathcal{I} \simeq 1.1$ for all three models.

5 Conclusion

In this letter, we have studied the ratio \mathcal{I} of Eq. (6) suggested in [4] to supply a better way to measure the exclusive rare decay $B \rightarrow K^*\gamma$. This ratio is free of uncertainties in the wave function in the BSW model in the SU(3) flavor symmetry. However, the spin treatment in this model is not relativistic. We have investigated the relativistic models [7, 8, 12, 13] and shown that these models even with very different wave functions reduce to the BSW model in the heavy b -quark limit. Thus in such models, the ratio \mathcal{I} is also 1, in the limits of heavy b -quark and SU(3)-flavor symmetries, independent of uncertainties in the wave function. Numerically, using the physical masses and different wave functions, the ratio \mathcal{I} still stays close to 1. Therefore, a direct measurement of the q^2 -spectrum of the semileptonic decay can supply accurate information for $B \rightarrow K^*\gamma$.

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